Educ Stud Math (2011) 77:389–397
DOI 10.1007/s10649-011-9319-0

Signs for you and signs for me: the double aspect of semiotic perspectives

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Published online: 7 April 2011

Abstract The comments below are meant to show that considerations of public and private realms and the tension between these realms arise in a natural and persistent way in discussions connected with semiotics. In particular, they arise out of the themes of body and sociocultural mathematical meaning-making, which are recurring themes of the papers in this volume. The public–private dichotomy is related to other dichotomies such as those between outer and inner and collective and individual. For educators, such dichotomies are important in that they reflect the division between students' own inner and individual understandings of mathematical ideas and their functioning within a shared sociocultural world of mathematical meanings.

Keywords Public–private dichotomy · Semiotics

Every sane person lives in a double world, the outer and the inner world…
Charles S. Peirce (1931–1958, Vol. 5, p.487)

Writing a postscript especially for a collection such as this so rich in variety should, to some extent, be an opportunity to review the marvelous papers brought together in it, to see where we have arrived having read them; but more than that, it should try and identify common strands, if any, linking these papers. It is the latter that I mostly wish to take up here. In particular, I want to focus on a persistent tension in them between outer and inner or collective and individual or, as I prefer, public and private worlds—signs for you and signs for me.

But, before that, a word is in order about the breadth and, as I pointed out, the variety of the collection. For although this is not meant to be my main point, I do want to emphasize that this kind of breadth is an almost unavoidable characteristic of semiotic perspectives,

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even when they purport to be restricted to a single field such as mathematics education. This is true not only in modern semiotic thought but also in the very earliest explicitly semiotic writings. Already Augustine, for example, like Stoic writers before him, saw that the study of signs extended from natural signs to conventional signs, in both cases a sign having the very general definition of something that causes something else to come into our thinking as a result of itself (*De Doctrina Christiana*, II.1: "Signum est enim res…aliud aliquid ex se faciens in cogitationem venire"). This sense that the world of signs ranges from the material signs in natural bodies—smoke as a sign of fire (an example Peirce continued to use in speaking of an index)—to the signs that make human language, is what makes the world of signs a true world. And it is more than just a matter of signification in a world: signs are operative also in logical processes of inference by which we draw conclusions about the world—a point noted by the Stoics and one that was of supreme importance to Peirce (see Eco, 1986 and Arzarello & Sabena in this volume).

Thus, it is not all surprising that in turning our attention to mathematics education from a semiotic point of view, we should find papers emphasizing the body as a source of semiosis in various ways including the production of indices for abstract notions such as area (see Hegedus & Moreno-Armella's paper), papers that concern ways in which cultural signification becomes objectification, and papers concerning the development of mathematical thought in terms of the conscious production of symbols (Schubring). We would hardly expect so many different papers with so many different areas of immediate focus in a special issue dedicated, say, to algebra learning. Nor would we expect the same intense presence of philosophical considerations in an area other than semiotics (excluding perhaps the philosophy of mathematics education, which itself has become infused with semiotic considerations!). But because signs are ubiquitous in all our thoughtful and cultural life, semiotic approaches give rise to this broad range of philosophically informed themes in a natural and effortless way.

Nevertheless, the particular set of papers that make up this special issue do not represent all possible directions a semiotic approach can take: there are, in fact, two areas of emphasis, maybe three, that appear in paper after paper. These are (1) the role of body in semiosis, or, simply, embodied semiosis, in which I also include the subject of gesture; and (2) the social character of mathematical understanding. The third area is history, but the historical character of mathematical thought is so closely allied to its social character that I am willing, albeit hesitantly, to subsume the historical under the social.

These two directions beg the main question of this postscript, namely what common strands link these papers, assuming that there are common strands? Or is it just that there is the embodiment crowd and the social meaning-making crowd? Well, for anyone who has gone through the papers in this special issue, even cursorily, the answer to that is obviously no, if only for the simple reason that so many of the papers contain elements of *both* areas of emphasis—and that, sometimes quite explicitly, as in Radford and Roth's paper, whose title begins with the word "intercorporeality," the society of embodied beings. There is indeed a kind of continuity between these emphases. One can see that easily enough by whimsically juxtaposing the titles of the well-known books, *The Body in the Mind* (Johnson, 1987) and *Mind in Society* (Vygotsky, 1978)!

And yet, on the face of it, body and society appear to occupy two opposing extremes of human experience. At the one extreme, I may call society *my own* society, as I might call culture *my own* culture; however, society and culture are in essence *shared* aspects of human experience. They cannot be totally my own, a society or culture of one. So at this extreme, we have a public world where what is for you is for me. In contrast, body, my body, is never yours, unless you are my Siamese twin. Indeed, one recalls Aristophanes' amusing tale in Plato's *Symposium* (Plato, 1899–1906,189e–193d) about the "man-woman" (*androgunos*) who was a man and a woman rolled up into one body— four arms, four legs, and two faces on one head. Zeus, fearing their strength and ambition, separated them, causing them, subsequently, to yearn and search for their other half, and that yearning to be whole again is what we call “love.” Thus, the explanation of love itself, in Aristophanes' telling, is founded on the very separateness of our bodies. The separateness of our experience as embodied creatures also lies behind philosophical questions that young people love asking: “How do I know my sensations are like yours?” “How do I know that you feel this smooth surface or this sharp corner as I do?” So, body, while it is visible to all, is unsharable, fundamentally private.

But surely the story cannot end here. Body is, as I said, visible to all. This makes body, private though it may be fundamentally, not only what mediates between our own thoughts and our perceptions and experience of the world, but it is also what mediates between our thoughts and others in the form of gestures. And it can be argued, as it has been here and elsewhere (Lakoff & Núñez, 2000, for example), that the *chief* mediator between ourselves and others, namely language itself, is informed by body. On the other side of the coin, society does not relate utterly and exclusively to the public world. For as we learn from so many of the papers in this special issue and certainly from Vygotsky, whose spirit hovers behind and within so many of them, our own sentiments, thoughts, and ways of thinking have sources in internalized social relations and socially engendered systems of meanings, semiotic systems. In fact, one might turn the tables and say that the bodily material side of things belongs to the public world of percepts, while internalized social relations are the atoms of our private world of imagination and ideas.

So, what we see is that body is somehow private, but also public, while society and culture are public, but also private—each extreme from opposite directions stretches toward and pulls away from a private–public center. What I want to underline is that that center is truly a place of tension and opposing forces more than it is a place where differences are resolved—and this tension is as much part of human experience as the public and private themselves are. And considering those book titles again, *The Body in the Mind* and *Mind in Society*, it may be that one's mathematical mind is located within that center of tension.

Of course, these kinds of considerations can arise in contexts other than semiotic ones; however, when the subject is signs, public–private considerations are unavoidable since signs signify for us and for others, and that, usually at the same time. But the public–private tension also speaks to all aspects of mathematics education where we want to balance students' own understanding with their understanding of shared bodies of knowledge and shared traditions (see Fried & Amit, 2003; Fried, 2008). What I would like to do then is simply elaborate on these points, show some ways in which the papers in this special issue reflect the public–private dichotomy, and in doing so clarify the importance of the dichotomy for mathematics education while showing its intimate connection to semiotic oriented research. I will begin with body, embodiment, and gesture, keeping in mind that this is only a matter of emphasis and that, as I remarked, many of the papers touch on both embodiment and social perspectives as one.

I should be clear that when I say “body,” I am using the word both as Johnson (1987) uses it, namely, “…as a generic term for the embodied origins of imaginative structures of understanding…” (p. xv), as well as a generic term referring to the bodily channel for the production and use of signs. With that in mind, body can provide a *general* framework for thinking about the learning of mathematical ideas. One such general framework is certainly that in Jennifer Thom and Michael Roth's paper, “Radical embodiment and semiotics: toward a theory of mathematics in the flesh.” Theirs is a true marriage of general ideas about embodiment and specifically semiotic ones, for they see body not so much with an eye to the knowledge and conceptualizations that flow from it, as in Johnson (1987) or, for that matter, in many of the early empiricists, as much as "…the ground of everything and anything we can signify" (this issue), that is, body as the "last signifier, the limit of the signifier…" (Thom and Roth, above, quoting Nancy, 1993). Their radical embodiment is radical because it sees body as the *principal* semiotic organ. Thom and Roth present their own example of the body-as-semiotic organ in their description of how an 8-year-old boy, Owen, compares and distinguishes two rectangular prisms. Hegedus and Moreno-Armella in their paper on "The emergence of mathematical structures," do not ascribe to the "mathematics in the flesh theory" explicitly; however, aspects of their work confirm it beautifully. For example, using the PHANTOM Omni—a machine that can transform mathematical entities into haptic experiences—they were able to construct a learning situation where young students (and adults) could think about the constancy of area of a triangle on a given base and having a given height in terms of the constancy of force felt by them as they dragged a vertex parallel to the base (this issue) (the Omini was set so that the force was proportional to the area of a triangle). Their account makes a persuasive case for the force itself becoming a signifier[1] and the basis for the participants' discussions about the mathematics in the learning situation.

Beyond these more general embodiment frameworks is the specific topic of gesture. Several of the papers above treat gesture one way or another—certainly Thom and Roth's paper does—but two of the papers concern gesture as their explicit theme. These are Lulu Healy and Solange Hassan Ahmad Ali Fernandes' paper "The role of gestures in the mathematical practices of those who do not see with their eyes," and Zurina Harun and Julian Williams' paper, "Gesturing for oneself," whose title, I confess, suggested my own. Gesturing does illustrate concretely the doubleness in semiotic perspectives, for while my gestures are, in an immediate way, signs for you, they are also signs for me.

Thus, Harun and Williams, while recognizing the social communicational aspects of gesture, emphasize that gesture may also be a reflection of inner, and I would add, private thought. They write, "…the point is that, if there is inner speech, that is a linguistic component of their inner communication, we cannot hear it; we only observe the gestural component of it in these cases" (this issue). This is really a central point in embodiment: our own thought is contained, radically, in our bodily existence; to gesture is then in some way to reveal our thought, perhaps even better than our words. Because our bodies are our own and our bodily experience is essentially our own as well, reading someone else's gestures is like trying to read someone's expressions, trying to get at someone's private thoughts, the kind of thing one "gives a penny" for.

Healy and Fernandes' paper on blind students is in many ways the most fascinating paper in the collection. Like Harun and Williams, Healy and Fernandes show how gestures reveal inner mental life, but also how they are communicative. They summarize their results by saying:

> We have argued that, in terms of the production of mathematical meanings, gestures serve as a means to create embodied abstractions associated with the identification of definition of general properties and relationships experienced as concrete in a particular setting but applicable beyond it. In embodied abstractions, this generality is communicated (to other people, but perhaps most notably to the individual who creates the gesture) bodily, the motor actions associated with the hand movements involving an activity, or a replaying of an activity with physical artefacts or processes. (this issue)

[1] One might also take this as an instance of Peirce's "secondness," and the thoughts arising from it Peirce's "energetic interpretant," but I will not pursue this.

That the students spoken of are *blind* is the key point here: a visual mode of communication is odd for someone who does not dwell in a visual world, unless their visual world does not depend on seeing an outer world. The visual world of the blind must be an internal world and their gestures an expression of that internal world. But the gestures are indeed communicative, and that for others who, unlike the blind students, can see them. The gestures described by Healy and Fernandes thus take on a truly double character: they are the embodiment of the blind students' internal vision, and they are signs not only for the blind students but also for those sighted people in their shared cultural milieu.

The way signs that have meaning for sighted people are also the embodied expressions of the blind students' understandings of mathematical ideas edges us into the second and predominant theme of these papers, namely the social constitution of mathematical understanding through the agency of signs. But before I turn to those papers, I would like to make a slight digression.

Healy and Fernandes refer to memory at least twice in their paper: once, for example, when they tell us that blind students tend to remember the steps of their solution strategies rather than record them (this issue), and once toward the end where they speak of blind students' gestures as "signs of *imagined re-enactions* of previous doings…" (this issue). Memory is not only at the heart of the internal world of the blind, but of the sighted as well. And more importantly for this postscript, it is at the intersection between the private and the public. On the private side, we look inward to explore our memory; the ability for someone else to probe one's memory is violation of one's private life; and losing memory is a loss of one's own sense of self. Memory is private in the same way that body is. But our memories are also in a deep way public and cognitive because they are places where socially constituted signs are at work. In this sense, memory can participate in a kind of shared consciousness the way written texts do. In our desperate fear of rote learning in mathematics and elsewhere, I think we have forgotten this possibility of memory, a possibility which was a commonplace in intellectual circles of the middle ages. Mary Carruthers, in her account of memory in the middle ages (Carruthers, 1990), thus says that, "…medieval culture [and I would certainly emphasize this word "culture" here] was fundamentally memorial, to the same profound degree that modern culture in the West is documentary" (p. 8), and she reminds us that, "…signs make something present to the mind by acting on memory" (p. 222). This role of memory as a cognitive center and place of inner commerce of signs, which was so clear in the middle ages, has been almost entirely ignored in mathematics education. It has not been ignored, however, by mathematicians reflecting on their own work. Stanislav Ulam, for instance, said in his autobiography (though not necessarily with a semiotic emphasis) that "good memory—at least for mathematicians and physicists—forms a large part of their talent" (Ulam, 1983, p. 181).It seems to me that memory deserves much more attention in semiotic research in mathematics education then it has received until now. But let us move on.

Relying on Leont'ev and Vygotsky's theoretical outlooks as well as their previous work, Radford and Roth provide an impressive and persuasive view of how mathematical meaning is a sociocultural process in their "Intercorporeality and ethical commitment: an activity perspective on classroom interaction." In describing Leont'ev's activity perspective, Radford and Roth write:

> …a distinctive trait of activity is that it is stimulated by the pursuit of a *collective* endeavor. Activity conceived of as joint pursuit runs against the individualistic views of the social and the individual and is articulated around what Leont'ev called the *object* of activity—that is to say, the *intentional object* to which all the individuals'

> efforts are directed. According to Leont'ev, this object appears twice: in material and ideal (reflected) form. (this issue, all emphases in the original)

To catch this social creation of both object and subject, and therefore in a mathematical setting, the social production of mathematical meaning, Radford and Roth coin the delightful and revealing term, "togethering." Central to Radford and Roth's idea of "togethering" is the "ethical commitment to [that] common cause" (p. 15). This necessity of commitment and obligation in social settings is often overlooked in sociocultural approaches to mathematics education, which tend to stress only *how* meaning may be constituted in a social setting. But in argumentation theory (e.g. Patterson & David Zarefsky, 1983), terms of commitment, such as the "burden of rejoinder" are basic. It is such commitment that makes a classroom into a full society of learners. Falk Seeger, in his paper "On meaning making in mathematics education: social, emotional, semiotic," argues in a way completely parallel to Radford and Roth. He too speaks about a "shared intentionality," and he also speaks of the kinds of affective aspects of learning in a society, for example, empathy.

But the more one thinks about the issues raised in these papers, so seemingly settled in the realm of the social, the more one begins to see intruding that same division between an outer and inner world, between the shared material and reflected ideal objects. We must not forget that even a socially engendered subjectivity is nevertheless subjectivity, a development of a self not only with others but also *apart* from others. It is for this reason that one must speak about empathy and cooperation in the first place. It is also for this reason—this otherness of others—that newness can emerge from social activity. Radford and Roth make this clear when they write:

> In the teaching-learning process [the Russian, *obuchenie* activity] as we conceive it here, neither participant can anticipate the precise nature of others' actions; the *obuchenie* activity has emergent qualities as it unfolds in unforeseeable ways. For the students, the result of joint activity is the emergence of the object, whereas for the teacher it is continuous unforeseeable repositioning of the object so that it becomes an object of consciousness for the students (this issue).

The division between outer and inner, the social and the individual, and public and private is not lost on Radford and Roth, nor on Godino, Font, Wilhelmi, and Lurduy for whom the distinction between the "personal, or idiosyncratic character of practices (personal practices) and the institutional one (social or shared practices)…" (this issue) is central in their ontosemiotic approach to mathematical practice. Seeger too is very much aware of these divisions—and not just their mere existence but also the true doubleness they express. Thus, he points, for example, to "…an apparent paradox in the fact that culture is meant to be typically implicit or even tacit on the one hand, and culture being shared and public…" (p. 6), and then later on the same page, "[the contradiction between learning as a public participation of discourse and learning as an implicit or indirect process of enculturation] expresses, so to speak, two complementary positions of theorizing: the exteriority of mind and interiorization as a developmental process." And in fact, the tension between the poles of social and private, public and private, is implicit in Leont'ev's own activity theory as Radford and Roth make abundantly clear by quoting his saying that "[Activity] is an overarching unit that constitutes an evolving and dynamic space of joint action 'containing in itself those internal, impelling contradictions, dichotomies, and transformations' that create the conditions for consciousness and the self to emerge" (this issue).

What I would want to stress in addition is only that the division between public and private is not one that we should seek to wash away, but to bring out, highlight, and embrace. In another, not an explicitly semiotic, work with Miriam Amit (Fried & Amit, 2008), we tried to show how two eighth-grade girls' thinking about the idea of proof was drawn between two poles, which we coded as “we” and “they”: for the girls, Yana and Ronit, providing an *argument* was something one did for oneself, perhaps in one's head, while providing a *proof* was something for the teacher, a private act versus a public act. The point was not that Yana and Ronit would ultimately have to settle on one side or the other. On the contrary, it was that they should begin to realize that, mathematically, they must live between both poles, that they should come to accept that learning in mathematics occurs within the tension between “we” and “they.” It is only in this regard that I slightly disagree with George Santi's very interesting analysis of Laura's difficulties with the tangent concept in his paper, “Objectification and semiotic function.” He certainly notes a tension analogous to that of Yana and Ronit:

> Laura lives a cognitive conflict when she tries to put together her personal meaning of tangent, what she learned in calculus, the unicity and existence of the tangent. Her confusion reflects disconnected mathematical reflexive activities she has been exposed to, that she has not harmonized in a unitary mathematical meaning. (this issue).

Learning, I maintain, should not culminate in a harmonization, final and static, but in harmonizing, something in motion.

But that difference aside, I thoroughly agree with Santi's understanding that “The sense-giving students are involved in can be seen as a convergence of the cultural meaning with the personal meaning” (this issue), where both sets of meanings as well as the process of converging itself are mediated by signs. The cultural meaning of course, as Santi makes clear throughout his paper, is also an historical one. History is a major theme in several of the papers, prominent among them, Gert Schubring's “On how *n* and *i* turned out to become indices in mathematical sequences and formulae,” Tony Brown and David Heywood’s “Geometry, subjectivity and the seduction of language,” and to a lesser extent Michael Otte’s “Evolution, learning and semiotics from a Peircean point of view.”

Schubring's paper is a masterful discussion of how the algebraic understanding of mathematics is guided by the process of symbolization. Interestingly enough, though in connection with geometry, Brown and Heywood also look at algebraization; however, they see a very different role of this process than Schubring does: where Schubring sees in algebraization, development in mathematical thinking, they see a process of constraining human possibilities. They base their argument (and their title!) on Husserlian ideas. Yet, in some ways, they move to one side of Husserl's main idea, namely that of sedimentation (see Husserl, 1936). For sedimentation is really double-edged: it is on the one hand the concealment of original meanings under layers of conceptualization, but in the very process of concealment, it is *also,* on the other hand, what allows for the power of algebra and, generally, of symbolic mathematics. Thus, Jacob Klein has written, “…this kind of forgetfulness [which sedimentation somehow always is, according to Husserl] accompanies, of necessity, the development and growth of science” (Klein, 1985, p. 77).

But if sedimentation is a necessary component of the development of science, in Husserl’s view, so is the necessity, and ability, to “reactivate” the original sources of meaning. This attempt to dig beneath sedimented meanings and reactivate the beginnings of our knowledge is what the historical enterprise in its deepest manifestation obliges us to do. To view mathematics historically, then, means, first of all, realizing its sedimented state. That realization becomes immediately the first step toward the reactivation of the original foundation of mathematical meaning. But that realization is our own realization, in a way, our own self-discovery, even if it be a self-discovery that can never be truly completed. So looking at mathematics historically places us actively and constantly between the culturally set forms of mathematics, its sedimented character, and our own self-discovery. History then again brings us to a point poised between a public mathematical world, whose public character comes precisely from the sedimented cultural meanings that make up its discourse, and a private world in which we discover our own ability to reactivate original intuition and, therefore, discover a personal mathematical meaning. History may even be the best path to make one cognizant of this doubleness with its constant emphasis on culture on one side and idiosyncrasy on the other (see Fried, 2007).

To sum up, my goal in these comments has not been to show that all the insights in all these papers reduce to the public–private dichotomy. Of course not. As I discussed at the start of this postscript, the semiotic approach breeds a tremendous variety of ideas—and these papers prove that point very well. What I did want to show was that considerations of public and private and the tension between them arise in a natural and persistent way in discussions connected with semiotics; in particular, I wanted to show they arise out of the themes of body and sociocultural perspectives which predominate the papers in this volume. Conversely, one should expect semiotic treatments of mathematics education to take into account the possibility of thinking quietly in the closed space of one's room as well as shared discourse within a given culture, a private world and a public world. Learners of mathematics are drawn into this public–private dichotomy almost from the very start of their studies. For they are expected to master mathematical practices common to some institutional framework—a school or university—or, more importantly, belonging to a given mathematical tradition; at the same time, they are expected to develop their own mathematical understanding: signs in a mathematical discourse must stand to *them* for something in some respect or capacity, as Peirce would say. In both directions, their learning and their general mathematical thinking are mediated by signs: these are what I called in my title, "signs for you" and "signs for me." I also wanted to emphasize that this doubleness is not something to remove or resolve, but to keep active, for one's mathematical identity is formed at the point of tension between the public and private worlds of mathematics.

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